TABLE III

Strip Width (mm)	ϵ_{eff} (st)	ϵ_{eff} (0)	$\epsilon_{eff}(w)$ at 4 GHz	8 GHz	12 GHz	16 GHz
Fused Quartz						
3.00	3.25	3.26	3.28	3.31	3.36	
1.50	3.06	3.09	3.10	3.12	3.15	
1.00	2.96	2.97	2.98	2.99	3.02	3.06
0.50	2.81	2.86	2.87	2.88	2.89	
0.15	2.63	2.66	2.67	2.67	2.68	
Alumina						
2.00	8.30	7.85	8.06	8.31	8.60	
0.90	7.63	7.07	7.27	7 40	7.60	
0.58	7.33	6.86	6.93	7 11	7.31	7.52
0.20	6.76	6.25	6.30	6.40	6.50	
0.07	6.41	6.00	6.00	6.10	6.20	

ment with the static theory (Fig. 4). The orientation of the crystallites depends on the manufacturing processes and is not necessarily constant over the substrate. The anisotropy in alumina substrates can be inconvenient, especially when used for circuits comprising narrow-band filters and when experimentally verifying theories.

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The Green's Function for Poisson's Equation in a Two-Dielectric Region

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Abstract—**The validity of the reciprocity relation satisfied by the Green's function for Poisson's equation in a two-dielectric region is briefly discussed.**

Introduction

In calculating the parameters of a stripline by variational techniques it is often necessary to determine first a Green's function for the two-dimensional Poisson's equation in the region bounded by the two conductors [1], [2]. Contrary to the case of a single dielectric [2], the symmetry properties of the Green's function in a two-dielectric region do not appear to have been dealt with in the literature.

The aim of this short paper is to point out that the reciprocity relation satisfied by the Green's function in the latter case is only valid for a specific form of the right-hand side of the differential equation defining the Green's function. Only the Green's function subject to Dirichlet boundary conditions will be considered.

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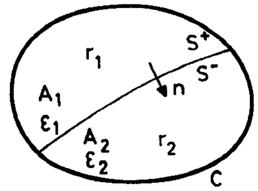

Fig. 1.

Reciprocity Relation

Let $G(r, r_0)$ be the function satisfying the following conditions in the two-dielectric region $A = A_1 \cup A_2$ (see Fig. 1):

$$\nabla^2 G = -\frac{1}{\epsilon}\delta(r - r_0), \qquad r \in A \tag{1a}$$

$$G = 0, \qquad r \in C \tag{1b}$$

$$G|_{S^+} = G|_{S^-} \tag{1c}$$

$$\epsilon_1 \frac{\partial G}{\partial n}\Big|_{S^+} = \epsilon_2 \frac{\partial G}{\partial n}\Big|_{S^-} \tag{1d}$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

Applying the Green's identity [2] separately to regions A_1 and A_2, in which G and its first-order partial derivatives are continuous with the only exception of the source point $(r = r_0)$, the following equations are readily obtained:

$$\frac{1}{\epsilon_1} G_2|_{r=r_1} = \int_S \left(G_1 \frac{\partial G_2}{\partial n} - G_2 \frac{\partial G_1}{\partial n}\right)_{r \in S^+} dl \tag{2a}$$

$$-\frac{1}{\epsilon_2} G_1|_{r=r_2} = -\int_S \left(G_1 \frac{\partial G_2}{\partial n} - G_2 \frac{\partial G_1}{\partial n}\right)_{r \in S^-} dl \tag{2b}$$

where G_1 and G_2 denote $G(r, r_1)$ and $G(r, r_2)$, respectively. Substitution of (1c) and (1d) into these equations yields the reciprocity relation

$$G(r_1, r_2) = G(r_2, r_1) \tag{3}$$

which shows that the Green's function is symmetric in its two arguments. Examination of (2a) and (2b) shows, however, that if the RHS of (1a) is simply $\delta(r - r_0)$, the reciprocity relation (3) no longer holds. In fact, it can be shown without much difficulty that in this case, the function G is not the true Green's function for Poisson's equation subject to the boundary conditions (1c) and (1d).

Finally we note that in view of relation (3), to determine G completely it is sufficient to consider the case where the source point is located in one of the two-dielectric regions, e.g., A_1.

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"Unfolding" the Lange Coupler

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The broad-band microstrip quadrature coupler described by Lange [1] is shown in Fig. 1(a). True quadrature coupling over an octave is realized as a consequence of the interdigital coupling section which compensates for even- and odd-mode phase velocity dispersion over the wide frequency range. A power-split variation between the direct and coupled ports, ports 3 and 4, respectively, in Fig. 1(a), of

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